MATH 10C: Calculus III (Lecture B00)

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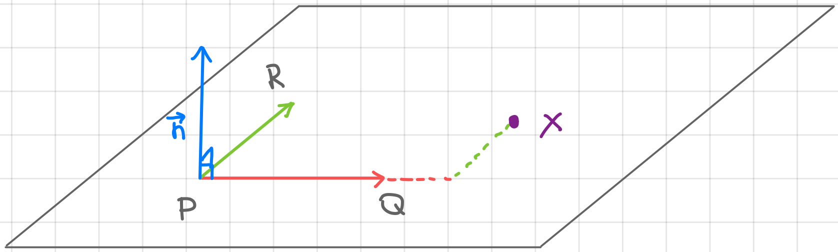
Today: Vector-valued functions

Next: Strang 3.2

Week 3:

- homework 3 (due Tuesday, October 18)
- Midterm 1: Wednesday, October 19 (vectors, dot product, cross product, equations of lines and planes)

Equation of a plane



Consider a plane containing point $P = (x_0, y_0, z_0)$ with normal vector $\vec{n} = \langle a, b, c \rangle$. Then point $X = (x, y, z)$ belong to this plane if and only if

$\vec{n} \perp \overrightarrow{PX}$, i.e. $\vec{n} \cdot \overrightarrow{PX} = 0$ vector equation of a plane

(*) $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$ scalar equation of a plane

If we denote $d := -ax_0 - by_0 - cz_0$, then (*) becomes

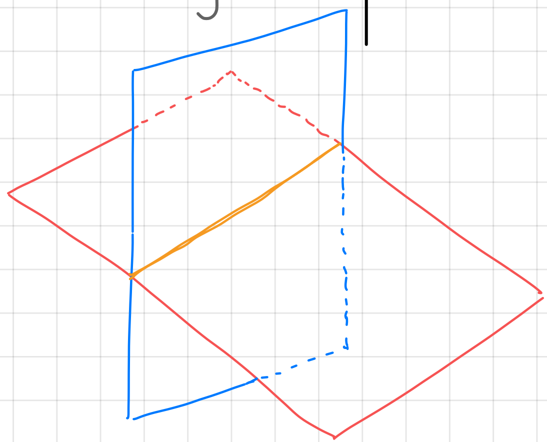
$ax + by + cz + d = 0$ general form of the equation of a plane

Parallel and intersecting planes

Let P_1 and P_2 be two planes in $\mathbb{R}^3$. Then the following possibilities exist:

		P_1 and P_2 share a common point: YES	P_1 and P_2 share a common point: NO
Normal vectors of P_1 and P_2 are parallel	YES	Equal	Parallel but not equal
	NO	Intersecting	

If two planes intersect, the intersection is a line!



Finding the line of intersection for two planes

Find the parametric and symmetric equations for the line formed by the intersection of the planes

$$x + 2y + 3z = 0 \quad , \quad x + y + z = -1$$

Vector-valued functions

<u>Definition</u> A vector-valued function is a function that takes real numbers as inputs and gives vectors as outputs, i.e.,

$\vec{r}(t) =$

$\vec{r}(t) =$

<u>Example</u> $\vec{r}(t) =$

$\vec{r}(t) =$

<u>Remark</u> From now on we will not distinguish between the point (x, y, z) and the vector $\langle x, y, z \rangle$, both are just lists of three real numbers

Vector-valued functions

Vector valued function $\vec{r}(t)$ often represents a

Think about the motion of a planet, flight of an airplane or a bird etc.

A vector-valued function may not be defined for all real numbers. For example, $\vec{r}(t) =$

is not defined for

You can explicitly specify the set of real number for which you want to define the function by writing, e.g.,

. We call this set the

Vector-valued functions

If the domain is not explicitly specified, we assume that it is the set of

Example

$$\vec{r}(t) = \left\langle \frac{1}{t}, \frac{1}{\cos t}, t \right\rangle$$

$$\operatorname{dom}(\vec{r}(t)) =$$

Sometimes the domain is found from the problem setup. If the function describes the motion of a bird between time 0 and tim T, then the domain is

Velocity and acceleration

Imagine a particle moving (smoothly) through space.

Let

The velocity is the

It describes the

The acceleration is the

Mathematically, the velocity is the derivative

and the accelaration is

The derivatives are computed

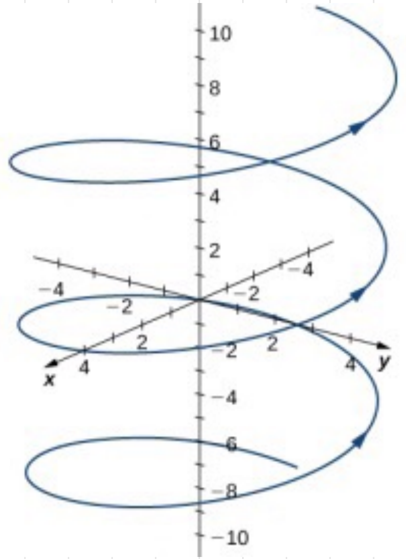
Velocity and acceleration

<u>Example</u> Let $\vec{r}(t) =$

The velocity :

The acceleration :

The path of this particle is called a



Limits of vector-valued functions

Let $\vec{r}(t) = \langle r_1(t), r_2(t), r_3(t) \rangle$ be a vector-valued function and let $\vec{L} = \langle L_1, L_2, L_3 \rangle$. Then the expression

means that

If one or more of the limits $\lim_{t \to t_0} r_1(t)$, $\lim_{t \to t_0} r_2(t)$ or $\lim_{t \to t_0} r_3(t)$ do not exist, we say that $\lim_{t \to t_0} \vec{r}(t)$ does not exist.

Example What is

$$\lim_{t \to 0} \left\langle \frac{\sin t}{t}, e^t, \cos t \right\rangle$$

Continuity of vector-valued functions

A vector-valued function $\vec{r}(t)$ is continuous at t_0 if

This is equivalent to $\lim_{t \to t_0} r_1(t) = r_1(t_0)$, $\lim_{t \to t_0} r_2(t) = r_2(t_0)$ and

$\lim_{t \to t_0} r_3(t) = r_3(t_0)$

Therefore, $\vec{r}(t)$ being continuous at t_0 is equivalent to

We say that $\vec{r}(t)$ is continuous if it is continuous at every single point t_0.

Derivatives of vector-valued functions

The derivative of a vector-valued function $\vec{r}$ is

$$\vec{r}\,'(t) =$$

provided that the limit exists. If $\vec{r}\,'(t)$ exists, we say that $\vec{r}$ is . If $\vec{r}$ is differentiable at every point t from the interval (a,b), we say that $\vec{r}$ is differentiable on (a,b).

Notice that if $\vec{r}(t) = \langle r_1(t), r_2(t), r_3(t) \rangle$, then

$$\vec{r}\,'(t) =$$

$$=$$

Calculus of vector-valued functions

Example Let $\vec{r}(t) = \langle \sin t, e^{2t}, t^2 - 4t + 2 \rangle$

Then

Summary

Calculus concepts (limit, continuity, derivative) are applied to vector-valued functions componentwise (apply to each component separately).

If $\vec{r}(t)$ represents the position of some object, then

- $\vec{r}\,'(t)$ is the velocity of this object ($\|\vec{r}\,'(t)\|$ is speed)
- $\vec{r}\,''(t)$ is the acceleration of the object

Integrals of vector-valued functions

Integration of vector-valued functions is done

if $\vec{r}(t) = \langle r_1(t), r_2(t), r_3(t) \rangle$, then

and if $a < b$

Example
- $\int \langle \sin t, t^2 + 2t, e^{2t} \rangle dt =$

$=$

- $\int_0^2 (\sin t \cdot \vec{i} + (t^2+2t) \cdot \vec{j} + e^{2t} \vec{k}) dt =$

$=$