MATH 10C: Calculus III (Lecture B00)

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Today: Vector-valued functions

Next: Strang 3.2

Week 4:

- homework 3 (due Tuesday, October 18)
- Midterm 1: Wednesday, October 19 (vectors, dot product, cross product, equations of lines and planes) MASKS !!!

Velocity and acceleration

Imagine a particle moving (smoothly) through space.
Let $\vec{r}(t)$ be its position at time t.

The velocity is the rate of change of the position
It describes the speed and direction of motion

The acceleration is the rate of change of the velocity

Mathematically, the velocity is the derivative $\vec{r}'(t) = \vec{v}(t)$
and the accelaration is $\vec{a}(t) = \vec{v}'(t) = \vec{r}''(t)$
The derivatives are computed componentwise, i.e.,

if $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$, then $\underset{\frac{d}{dt}\vec{r}(t)}{\vec{r}'(t)} = \langle f'(t), g'(t), h'(t) \rangle$

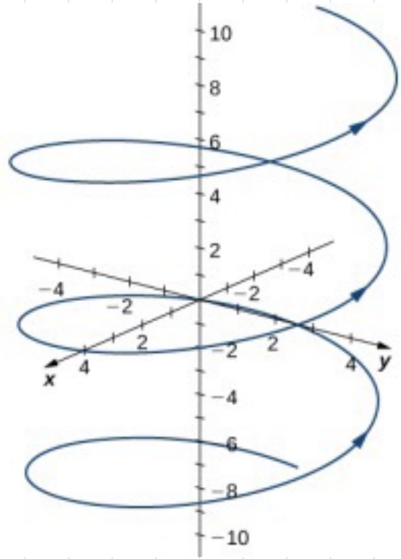
Velocity and acceleration

Example Let $\vec{r}(t) = \langle \cos t, \sin t, t \rangle$

The velocity: $\vec{r}\,'(t) = \langle -\sin t, \cos t, 1 \rangle$

The acceleration: $\vec{r}\,''(t) = \langle -\cos t, -\sin t, 0 \rangle$

The path of this particle is called a



Limits of vector-valued functions

Let $\vec{r}(t) = \langle r_1(t), r_2(t), r_3(t) \rangle$ be a vector-valued function and let $\vec{L} = \langle L_1, L_2, L_3 \rangle$. Then the expression

$$\lim_{t \to t_0} \vec{r}(t) = \vec{L}$$

means that

$$\lim_{t \to t_0} r_1(t) = L_1, \quad \lim_{t \to t_0} r_2(t) = L_2, \quad \lim_{t \to t_0} r_3(t) = L_3$$

If one or more of the limits $\lim_{t \to t_0} r_1(t)$, $\lim_{t \to t_0} r_2(t)$ or $\lim_{t \to t_0} r_3(t)$ do not exist, we say that $\lim_{t \to t_0} \vec{r}(t)$ does not exist.

Example What is

$$\lim_{t \to 0} \left\langle \frac{\sin t}{t}, e^t, \cos t \right\rangle = \langle 1, 1, 1 \rangle$$

$$\lim_{t \to 0} \frac{\sin t - 0}{t - 0} = \lim_{t \to 0} \frac{\sin t - \sin 0}{t - 0} = \sin'(0) = \cos(0) = 1$$

Continuity of vector-valued functions

A vector-valued function $\vec{r}(t)$ is continuous at t_0

if $\displaystyle\lim_{t \to t_0} \vec{r}(t) = \vec{r}(t_0)$

This is equivalent to $\displaystyle\lim_{t\to t_0} r_1(t) = r_1(t_0)$, $\displaystyle\lim_{t\to t_0} r_2(t) = r_2(t_0)$ and

$\displaystyle\lim_{t\to t_0} r_3(t) = r_3(t_0)$ holding simultaneously

Therefore, $\vec{r}(t)$ being continuous at t_0 is equivalent to all three components $r_1(t)$, $r_2(t)$, $r_3(t)$ being continuous at t_0.

We say that $\vec{r}(t)$ is continuous if it is continuous at every single point t_0.