MATH 10C: Calculus III (Lecture B00)

mathweb.ucsd.edu/~ynemish/teaching/10c

Today: Vectors in the plane

Next: Strang 2.2

Week 1:

- check the course website
- homework 1 (due Friday, September 30)
- join Piazza, Edfinity

Definition of a vector

We use scalars (numbers) to describe various quantities. Example: time, distance, mass, speed are represented by a single number (a scalar)

Certain quantities cannot be described by scalars. Think about the movement of an airplane. We need to know

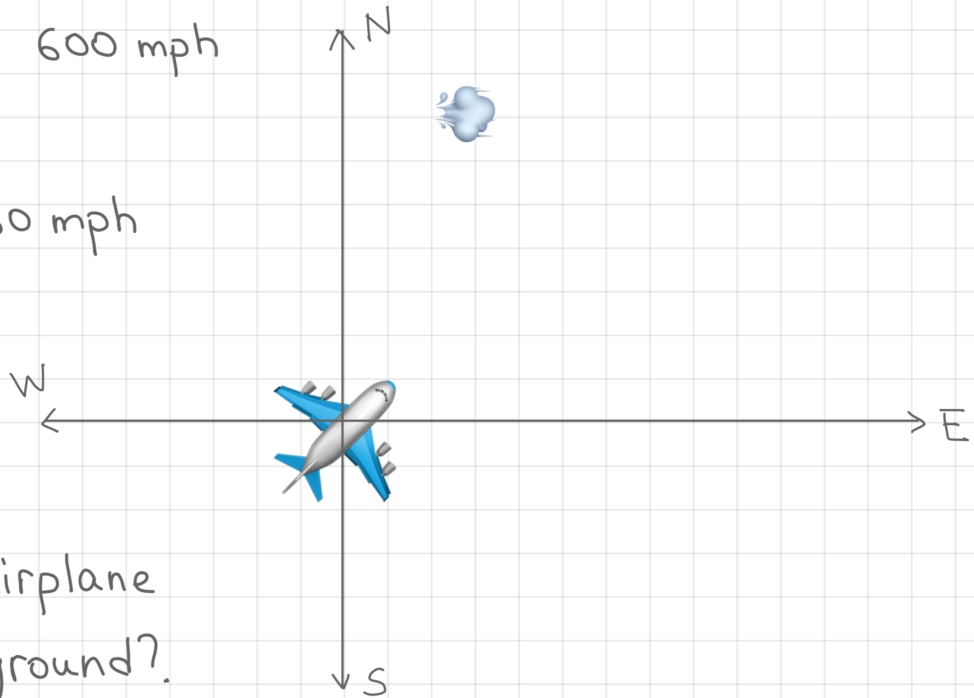
the direction of the movement of the airplane
the speed of the airplane

Definition of a vector

Forces, displacements, velocity are described by vectors.

Airplane flies NE at 600 mph
(relative to the air)

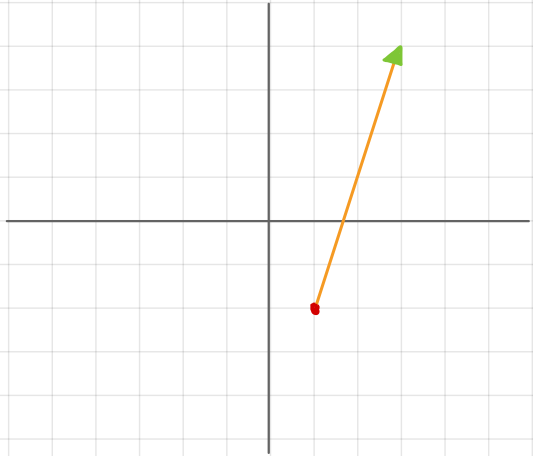
Wind blows SE at 60 mph



How fast does the airplane
fly relative to the ground?
In what direction ?

Initial and terminal points. Magnitude

A vector in a plane is represented by a directed line segment from the   to the



The length of the line segment represents the

Notation: vectors are denoted

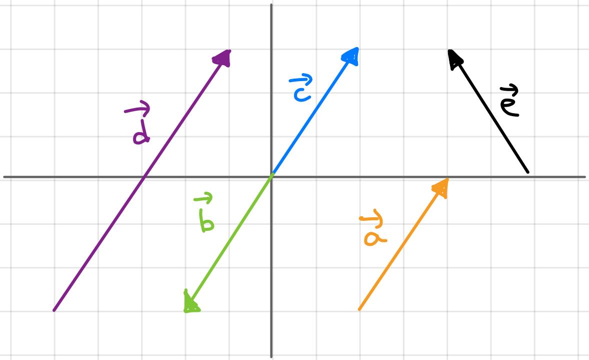
the magnitude of the vector is denoted

Zero vector. Equivalent vectors

A vector with an initial and terminal points that are the same is called

We say that $\vec{v}$ and $\vec{w}$ are if they have the same direction and magnitude

We treat equivalent vectors as equal even if they have different initial points.



$\vec{a}$ $\vec{b}$

$\vec{a}$ $\vec{c}$

$\vec{a}$ $\vec{d}$

$\vec{a}$ $\vec{e}$

Scalar multiplication

Let $\vec{v}$ be a vector and k be a real number

Then $k\vec{v}$, called

is a vector such that

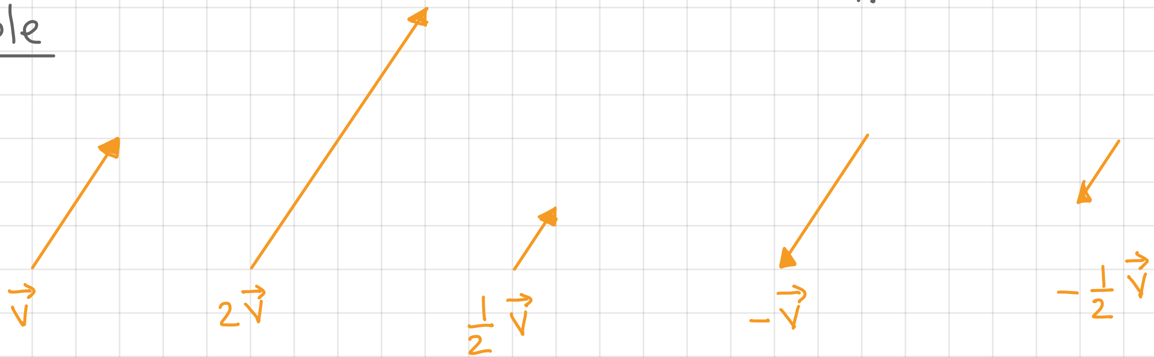
$\|k\vec{v}\| =$

$k\vec{v}$ has the as $\vec{v}$ if

$k\vec{v}$ has the direction to the direction of $\vec{v}$ if

If $k=0$ or $\vec{v}=0$, then

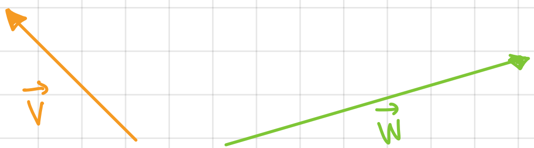
Example



Vector addition

Let $\vec{v}$ and $\vec{w}$ be two vectors. Place the initial point of $\vec{w}$ at the terminal point of $\vec{v}$. Then the vector with initial point at the initial point of $\vec{v}$ and the terminal point at the terminal point of $\vec{w}$ is called the , and is denoted

Example



Notice that

Combining vectors

We know how to define (geometrically) $k_1\vec{v}_1 + k_2\vec{v}_2$

or $k_1\vec{v}_1 + k_2\vec{v}_2 + k_3\vec{v}_3 \ \ldots$

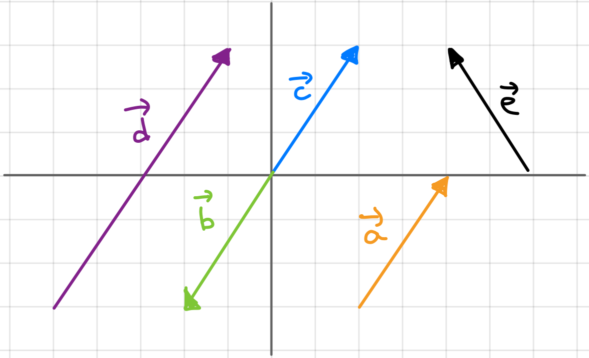
Example

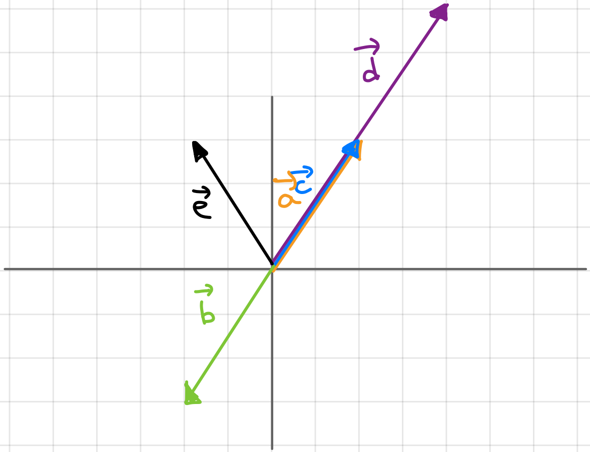
- $\vec{w} - \vec{v}$

- $2\vec{v} + \frac{1}{2}\vec{w}$

Vector components

It is easier to work with vectors in a coordinate system. Since the location of the initial vector does not matter, let's place all vectors in the plane so that their initial points



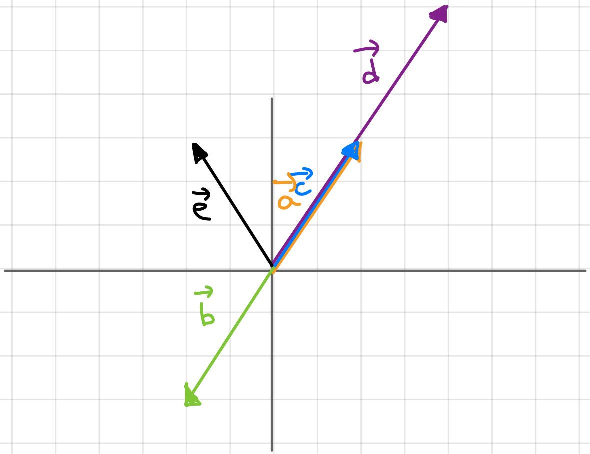


We call such vectors , and

they can be described by the

Vector components

<u>Def.</u> The vector with initial point $(0,0)$ and the terminal point (x,y) can be written in component form as . The scalars x and y are called the



$\vec{a} =$

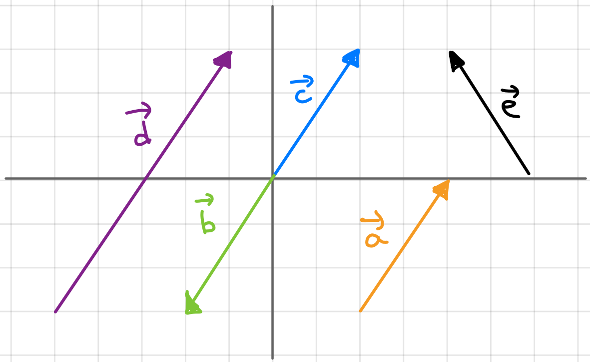
$\vec{b} =$

$\vec{d} =$

$\vec{e} =$

Vector components

If the vector is not in standard position, but we know the coordinates of its initial and terminal points, then we can find the vectors coordinates using the following rule:



Magnitude of the vector

Magnitude of the vector is the distance between its initial and terminal points.

If $P = (x_i, y_i)$, $Q = (x_t, y_t)$, then

If $\vec{v} = \langle x, y \rangle$, then

Example

- $P = (2, -3)$, $Q = (4, 0)$

- $\vec{a} = \langle 2, 3 \rangle$

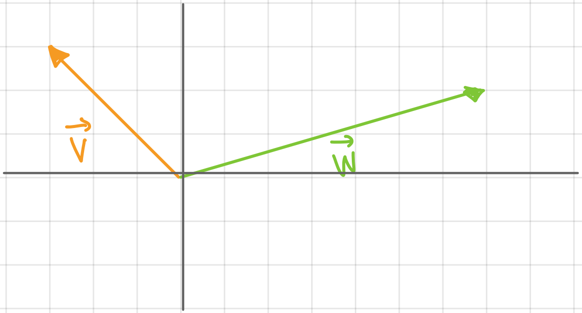
Vector operations in component form

__Def.__ Let $\vec{v} = \langle x_1, y_1 \rangle$, $\vec{w} = \langle x_2, y_2 \rangle$, $k \in \mathbb{R}$.

Then

- $k\vec{v} =$ (scalar multiplication)
- $\vec{v} + \vec{w} =$ (vector addition)

__Example__



- $\vec{v} + \vec{w} =$
- $\vec{w} - \vec{v} =$
- $2\vec{v} + \frac{1}{2}\vec{w} =$