

## MATH 180A HOMEWORK 2

WINTER 2023

Due date: **Sunday 01/29/2023 11:59 PM** (via Gradescope)

1. (*ASV*, *Exercise 2.2*) - 2 points.  
A fair coin is flipped three times. What is the probability that the second flip is tails, given that there is at most one tails among the three flips?
2. (*ASV*, *Exercise 2.4*) - 2 points.  
We have two urns. The first urn contains two balls labeled 1 and 2. The second urn contains three balls labeled 3, 4 and 5. We choose one of the urns at random (with equal probability) and then sample one ball (uniformly at random) from the chosen urn. What is the probability that we picked the ball labeled 5?
3. (*ASV*, *Exercise 2.6*) - 3 points.  
When Alice spends the day with the babysitter, there is a 0.6 probability that she turns on the TV and watches a show. Her little sister Betty cannot turn the TV on by herself. But once the TV is on, Betty watches with probability 0.8. Tomorrow the girls spend the day with the babysitter.
  - (a) What is the probability that both Alice and Betty watch TV tomorrow?
  - (b) What is the probability that Betty watches TV tomorrow?
  - (c) What is the probability that only Alice watches TV tomorrow?

*Hint:* Define events precisely and use the product rule and the law of total probability.
4. (*ASV*, *Exercise 2.10*) - 2 points.  
I have a bag with 3 fair dice. One is 4-sided, one is 6-sided, and one is 12-sided. I reach into the bag, pick one at random and roll it. The outcome of the roll is 4. What is the probability that I pulled out the 6-sided die?
5. (*ASV*, *Exercise 2.32*) - 3 points.  
Suppose the family has 3 children of different ages. We assume that all combinations of boys and girls are equally likely.
  - (a) Formulate precisely the sample space and probability measure that describes the genders of the three children in the order in which they are born.
  - (b) Suppose we see the parents with two girls. Assuming we have no other information beyond that at least two of the children are girls, what is the probability that the child we have not yet seen is a boy?
  - (c) Suppose we see the parents with two girls, and the parents tell us that these are the two youngest children. What is the probability that the oldest child we have not yet seen is a boy?

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\**Introduction to Probability*, by David F. Anderson, Timo Seppäläinen, and Benedek Valkó

6. (*ASV, Exercise 2.40*) - 3 points.

Incoming students at a certain school take a mathematics placement exam. The possible scores are 1, 2, 3, and 4. From past experience, the school knows that if a particular student's score is  $x \in \{1, 2, 3, 4\}$ , then the student will become a mathematics major with probability  $\frac{x-1}{x+3}$ . Suppose that the incoming class had the following scores: 10% of the students scored a 1, 20% of the students scored a 2, 60% scored a 3, and 10% scored a 4.

- (a) What is the probability that a randomly selected student from the incoming class will become a mathematics major?
- (b) Suppose a randomly selected student from the incoming class turns out to be a mathematics major. What is the probability that he or she scored a 4 on the placement exam?

7. (*ASV, Exercise 2.42*) - 2 points.

Urn  $A$  contains 2 red and 4 white balls, and urn  $B$  contains 1 red and 1 white ball. A ball is randomly chosen from urn  $A$  and put into urn  $B$ , and a ball is then chosen from urn  $B$ . What is the conditional probability that the transferred ball was white given that a white ball is selected from urn  $B$ ?

8. 3 points.

Consider the following variant of the Monty Hall problem. Suppose that you choose door 1. In the case that door 1 has the prize behind it, suppose that Monty opens one of the other two doors for you not uniformly at random, but rather opens door 2 with probability  $p$  and door 3 with probability  $1 - p$ , for some  $p \in [0, 1]$ . In the case that door 1 does not have the prize behind it, Monty opens the only remaining door without the prize behind it. How does this affect your chances of getting the prize when switching? For which values of  $p$  should you switch? To make your decision, you should calculate the probability that the prize is behind door 1 given that Monty opens door 2.